Allen G. Hunt's book "Percolation Theory for Flow in Porous Media", pp. 44-49, gives the continuous **probability density function (pdf)** for pores with radii in the interval r to $r + dr$, in a porous medium of fractal dimension D_p as:

$$W(r) = \frac{(3 - D_p)}{r_{max}^{3-D_p}} \cdot r^{-1-D_p} \quad \text{with} \qquad r_{max} \le r \le r_{min} \ . \qquad (1)$$

This global pdf is valid for the distribution of pores as a whole. Look ahead for the figure labeled "Hunt's pdf & Friends FD = 2.86" for an example of $W(r)$ for the case j = 0.20, D_p = 2.86, r_{max} = 0.20000 cm, and r_{min} = 0.04063 cm, in limestone. The curve for $W(r)$ is shown in magenta. Note the logarithmic scale for the y-axis: this indicates a tremendous increase as the pore radius r decreases towards r_{min} . **In this fractal porous medium, given an arbitrary pore, it is much more likely to be smaller r rather than large r – i.e. most pores in this fractal medium are small.**

Following Hunt, the total porosity j is just the integral of $W(r)$ times r^3 over the interval [r_{min} , r_{max}] and leads to the key result:

$$j = 1 - (r_{min} / r_{max})^{3-D_p} , \qquad (2)$$

which is consistent with results obtained by Rieu and Sposito using discrete methods. As Hunt remarks, $W(r)$ **and its normalization were chosen to achieve this result.** This pdf has units of probability per unit volume.

The **initial scenario for fractal porous media by the forward nuclear micro-geophysical model LVPM** assumes that laboratory measurements result in INPUT values for j , D_p , and r_{max}. Values for the "INPUT" r_{min} are then chosen to be consistent with (2). ***However, the exact details of the interface between the fractal quantities described here and LVPM are not provided at this time.***

Further flexibility and insight into the implications of (1) for a fractal porous medium can be obtained as follows. Break the pore radius interval [r_{min} , r_{max}] up into (say) ten smaller (equally spaced) intervals called <u>classes</u>:

$$r_{min}, \ \ r_{min} + (r_{max} - r_{min})/10 , \ \ r_{min} + 2(r_{max} - r_{min})/10 , \ldots , \ \ r_{max} . \qquad (3)$$

These classes are more closely attuned to laboratory measurements of fractal dimension for soils and rocks. For each pore size class in this sequence, compute the integral of $W(r)$: this is proportional to the probability that pores have sizes in that class:

$$\mathrm{Prob}_i = \int_{i,lo}^{i,hi} W(r)dr = A_0 \cdot \frac{(3-D_p)}{D_p \cdot (r_{\max}^{3-D_p})} \cdot (r_{i,lo}^{-D_p} - r_{i,hi}^{-D_p}) \ . \qquad (4)$$

The constant A_0 is determined by forcing the sum of the Prob_i over all classes to equal unity. The curve with red squares in this same figure shows these probabilities at a fractal dimension of 2.86.

For each pore size class in this same sequence, one can also compute the average porosity by integrating $W(r)$ times r^3 resulting in

$$f_i = \int_{i,lo}^{i,hi} W(r) \cdot r^3 dr = \left(r_{\max}^{3-D}\right)^{-1} \cdot (r_{i,hi}^{3-D_p} - r_{i,lo}^{3-D_p}). \qquad (5)$$

The average porosity for all ten classes at a fractal dimension of 2.86 is 0.02, with a standard deviation of 0.0085. Recall that the total porosity of this limestone is fixed at 0.20. This figure clearly shows that most of this porosity is contained in its smaller pores.

The reader is invited to look at all 22 figures in this sequence of fractal dimensions from 2.999 to 1.001, with a fixed $r_{\max}$ value of 0.20000 cm. As the fractal dimension decreases, note that $r_{\min}$ increases towards $r_{\max}$. At a fractal dimension of 2.00, all classes have the same porosity, namely 0.02. As the fractal dimension continues to decrease in its approach to 1.001, $r_{\min}$ continues to increase, albeit more slowly, and more and more of the porosity is located in the larger pores. Neither $W(r)$ nor its class integrals ever decrease as the pore size increases for fractal dimensions from 2.999 to 1.001.