TRANSMISSION PROBABILITY METHOD APPLIED TO LAMINATED VUGGY POROUS MEDIA

by
Mike Smith

Introduction

The Transmission Probability Method (TPM) is a “back to basics” computational technique within the LVPM program that provides a more detailed and more accurate description of the propagation of neutrons and gamma rays in porous media. TPM can interrelate pore size, pore shape, and laminae bed thicknesses with the mixing rules of all **neutron** macroscopic scattering and absorption cross sections; and the **gamma ray** mass attenuation coefficients, mass energy coefficients, and linear attenuation coefficients. Ultimately, using various logging tool proxy models, the LVPM program provides mixing rules for various formation physical parameters including porosity, capture cross section, bulk density, and Pe.

TPM converts the original experimental nuclear cross section data taken in various laboratories using (generally) thin, homogeneous samples with no pores - in both transmission and scattering geometries - into a form more useful for computing the response of nuclear logging tools in thick formations having pore systems with finite sizes. TPM also provides a definite, well-defined method for converting basic tool responses and shop calibrations recorded in media with/without pores into ones more suitable for logging real earth formations with finite pore sizes.

TPM does this by converting macroscopic cross sections and linear attenuation coefficients for homogeneous media into their corresponding values for heterogeneous media. TPM is not meant to imply that the wellbore geometry is a transmission geometry for the various nuclear logging tools (or that neutrons and gamma rays travel only as plane waves in the borehole-formation region).

Historical Perspectives

The original forward models SNUPAR and MSTAR from Schlumberger and Halliburton used nuclear cross section data bases to compute macroscopic absorption and scattering cross sections at various neutron energies and then applied these cross sections in their proxy models to obtain the response of neutron logging tools to formations composed of virtually any minerals and any fluids and also to help delineate departure curves within their chart books. Proxy models were calibrated using both limited experimental data and Monte Carlo calculations.

Doctor Neutron has extended the content and scope of these forward models in several very significant ways. These models used linear mixing rules to compute the macroscopic cross sections. Doctor Neutron demonstrated that linear mixing rules imply infinitesimal pore sizes. He then developed expressions for macroscopic cross sections for finite pore sizes with non-linear mixing rules and also extended the original scope of the forward models to include gamma ray mass and linear attenuation coefficients for a very wide range of gamma energies. Finally, Doctor Neutron extended the scope of TPM to handle laminated porous media.

The original forward models and those recently developed by Doctor Neutron have never assumed transmission geometries in their proxy models. The Transmission Probability Method simply develops new expressions for the macroscopic neutron scattering and absorption cross sections and gamma linear attenuation coefficients in order to describe heterogeneous porous media. For both the older and newer software, these cross sections and attenuation coefficients are used to drive the logging tools' proxies. Several unexpected results from this process have been the introduction of non-linear mixing rules for many quantities of physical interest, as well as pore size and pore shape effects and laminated bed thickness effects beyond what might be expected from classic bed thickness weighting.

Transmission Probability Method Details for a Single Vuggy Porous Medium

This method is applied here to the familiar thermal neutron capture(absorption) cross section in a vuggy porous formation, one that has a fairly straightforward pore geometry. Similar considerations apply to the linear attenuation coefficient for the gamma rays. In the larger context, the LVPM program (Laminated Vuggy Porous Media) applies this method to all the neutron and gamma processes at all energies for a pair of laminated vuggy porous media.

Assume that the pore system is clumped into inclusions of average volume $V[cm^3]$ and associated linear dimension $\lambda[cm]$ so that

$$V = \lambda^3 \qquad \textbf{(1)}$$

If the average porosity of the formation is ϕ, then each pore is associated on average with a formation volume V (i.e. Rock + fluid) given by

$$\mathrm{V} = V / \phi \qquad \textbf{(2)}$$

V also has an associated linear dimension L[cm] such that

$$\mathrm{V} = \mathrm{L}^3 \qquad \textbf{(3)}$$

Note that

$$\mathrm{L} = \mathrm{V}^{1/3} = \left(V / \mathrm{f}\right)^{1/3} \quad \textbf{(4)}$$

and

$$\boxed{\mathrm{L} = \mathrm{l} \cdot \mathrm{f}^{-1/3}} \quad \textbf{(5)}$$

V represents the minimum volume of formation that is meaningful to consider – it contains on average a single pore. It turns out that L plays a key role in the computerized implementation of all TPM algorithms, including those for parallel and perpendicular laminae. Note that L depends on **both** l and f !

With reference to Figure 1, consider the propagation of a beam of neutrons (or gamma rays) through our vuggy porous system. From <u>Neutron Physics</u> by Beckurts and Wirtz, pp. 3-5, the macroscopic absorption cross section for thermal neutrons sigma (Σ) is the probability per unit length of absorption and so the probability of survival/transmission to a microscopic distance Δx is $1 - \Sigma \cdot \Delta x$, where $\Sigma \cdot \Delta x << 1$. For a macroscopic distance x in a **homogeneous** porous medium, the transmission probability is $e^{-\Sigma x}$ and we can write

$$P_{HOMO}^{TRANS} = e^{-\Sigma x}. \quad \textbf{(6)}$$

In fact, we could just define Σ_{HOMO} by the expression

$$\Sigma_{HOMO} = -\ln(P_{HOMO}^{TRANS}) / x. \quad \textbf{(7)}$$

For any **heterogeneous** porous medium, we shall calculate the transmission probability P_{HET}^{TRANS} and then define the effective heterogeneous macroscopic cross section Σ_{HET} by the expression

$$\boxed{\Sigma_{HET} = -\ln(P_{HET}^{TRANS}) / x} \quad \textbf{(8)}$$

In order to calculate P_{HET}^{TRANS} to a macroscopic distance x in Figure 1, slice the formation up into (x/L) slabs of thickness L perpendicular to the neutron beam. L is the associated linear dimension (4,5). Each slab contains on average one pore in the x direction. Each slab is composed of vug/pore and rock, both of whose components are assumed to be homogeneous and characterized by macroscopic absorption cross sections Σ_{rock} and Σ_{vug}. It is important to understand that these cross sections imply respective porosities of 0 and 1. Each slab has cross sectional area to the beam S[cm^2] and so has volume SL and contains SL/V = SL(f / V) pores. Since each pore has an area to the beam $\mathrm{l}^2 = V^{2/3}$, the pores in each slab present an area to the beam

$$\mathrm{SL} \cdot (\mathrm{f} / V) \cdot V^{2/3} = \mathrm{SL} \cdot (\mathrm{f} / V^{1/3}) = \mathrm{S}\,\mathrm{f}^{2/3} \quad \textbf{(9)}$$

Then for a single slab the transmission probabilities are

$$\boxed{P_{rock}^{slab} = e^{-\Sigma_{rock} L} \cdot (S - S \cdot \mathrm{f}^{2/3}) / S = (1 - \mathrm{f}^{2/3}) \cdot e^{-\Sigma_{rock} L}} \quad \textbf{(10)}$$

and

$$P_{vug}^{slab} = e^{-\Sigma_{vug}\lambda} \cdot e^{-\Sigma_{rock}(L-\lambda)} \cdot S\phi^{2/3} / S$$

or

$$\boxed{P_{vug}^{slab} = \phi^{2/3} \cdot e^{-\Sigma_{rock}L} \cdot e^{-(\Sigma_{vug}-\Sigma_{rock})\lambda}} \qquad \textbf{(11)}$$

(In the expressions for P_{vug}^{slab}, the neutrons must penetrate both rock **and** vug.) Hence, the total transmission probability to a depth L in the heterogeneous medium is

$$P_{HET}^{slab} = P_{rock}^{slab} + P_{vug} = e^{-\Sigma_{rock}L}[1-\phi^{2/3}(1-e^{-(\Sigma_{vug}-\Sigma_{vug}))\lambda})], \qquad \textbf{(12)}$$

since the neutrons penetrate either the rock section **or** the rock-vug section. Then the total transmission probability to a distance x through (x/L) slabs is

$$P_{HET}^{TRANS} = [P_{HET}^{slab}]^{x/L} = e^{-\Sigma_{rock}x}[1-\phi^{2/3}(1-e^{-(\Sigma_{vug}-\Sigma_{rock})\lambda})]^{x/L} \qquad \textbf{(13)}$$

since the neutrons must penetrate slab1 **and** slab2 **and** slab3 …**and** slab(x/L). Then use of equation (8) yields

$$\Sigma_{HET} = \Sigma_{rock} - (1/L)\ln[1-\phi^{2/3}(1-e^{-(\Sigma_{vug}-\Sigma_{rock})\lambda})],$$

or, using equation (5)

$$\boxed{\Sigma_{HET} = \Sigma_{rock} - (\phi^{1/3}/\lambda)\ln[1-\phi^{2/3}(1-e^{-(\Sigma_{vug}-\Sigma_{rock})\lambda})]} \qquad \textbf{(14)}$$

This equation is the generalization of the mixing rule for a vuggy porous system at porosity ϕ, with absorption cross sections Σ_{rock} and Σ_{vug}, and with pore size λ. An equation similar to (14) appears on the website doctorneutron.com. It was first derived by Zakharchenko in 1967 using an entirely different approach.

Equation (14) satisfies a number of important limiting conditions when used with equations (1-5):

$$\begin{aligned}
&\text{(1) when } \phi \to 0,\ \Sigma_{HET} = \Sigma_{rock};\\
&\text{(2) when } \phi \to 1,\ \Sigma_{HET} = \Sigma_{vug};\\
&\text{(3) when } V \to 0,\ \Sigma_{HET} \to \Sigma_{rock}(1-\phi) + \Sigma_{vug}\phi .
\end{aligned} \qquad \textbf{(15)}$$

This last limiting condition means that the heterogeneous cross section continuously grades over into the homogeneous cross section as the pore volume tends to zero.

Because the derivation here is symmetric in rock/vug properties, a wide variety of 2 component mixture problems is actually supported: (1) pores within a rock matrix;

(2) pebbles within a fluid bath, (3) structural clay globs within another solid matrix, and (4) other mineral inclusions within a rock matrix.

Standard/Classic/Homogeneous Bed Thickness Weighting for Laminated Beds

The standard/classic/homogeneous approach generally uses **bed thickness weighting/mixing:** it forms the baseline for comparison with the newer LVPM method. Suppose Material1 (sand) has a thickness Δ_1 and Material2 (shale) has thickness Δ_2. The total thickness of one laminar cycle is $\Delta = \Delta_1 + \Delta_2$ and so the corresponding bed thickness weights are

$$W_1 = \Delta_1 / \Delta \text{ and } W_2 = \Delta_2 / \Delta . \qquad \textbf{(16)}$$

For the **neutrons**, all macroscopic cross-sections for all energies and for both absorption and elastic scattering processes obey simple bed thickness weighting.

$$\Sigma^i_{LAM} = W_1 * \Sigma^i_1 + W_2 * \Sigma^i_2 \qquad \textbf{(17)}$$

For the **gamma rays**, these rules are more complex even in the homogeneous case. Mixing rules for the linear attenuation coefficients (LAC) at various energies are central to both the LVPM homogeneous and heterogeneous calculations. For the gammas, tables of mass attenuation coefficients (MAC) from Hubbell and Seltzer are used and LAC is computed from the general relationship

$$LAC = \rho * MAC , \qquad \textbf{(18)}$$

where ρ is bulk density. Bed thickness weighting is applied to ρ and MAC as follows. Total Material1 and Material2 atomic weights are computed and so the bed thickness average atomic weight is

$$AverageAtomicWeight = W_1 * AtomicWeight_1 + W_2 * AtomicWeight_2 \qquad \textbf{(19)}$$

Then the Material1 and Material2 mass fractions are

$$MassFraction_1 = W_1 * AtomicWeight_1 / AverageAtomicWeight \qquad \textbf{(20)}$$

and

$$MassFraction_2 = W_2 * AtomicWeight_2 / AverageAtomicWeight . \qquad \textbf{(21)}$$

These mass fractions are used to compute mass attenuation coefficients at all gamma energies. The final gamma ray mixing rule needed for the classic/homogeneous LVPM computation of the linear attenuation coefficients involves the bulk density itself:

$$\rho_{LAM}^{HOMO} = W_1 * \rho_1 + W_2 * \rho_2 . \qquad \textbf{(22)}$$

This rule follows from a proper accounting of the masses present in the sand and shale laminae, assuming their materials are homogeneous with infinitesimal pore sizes.

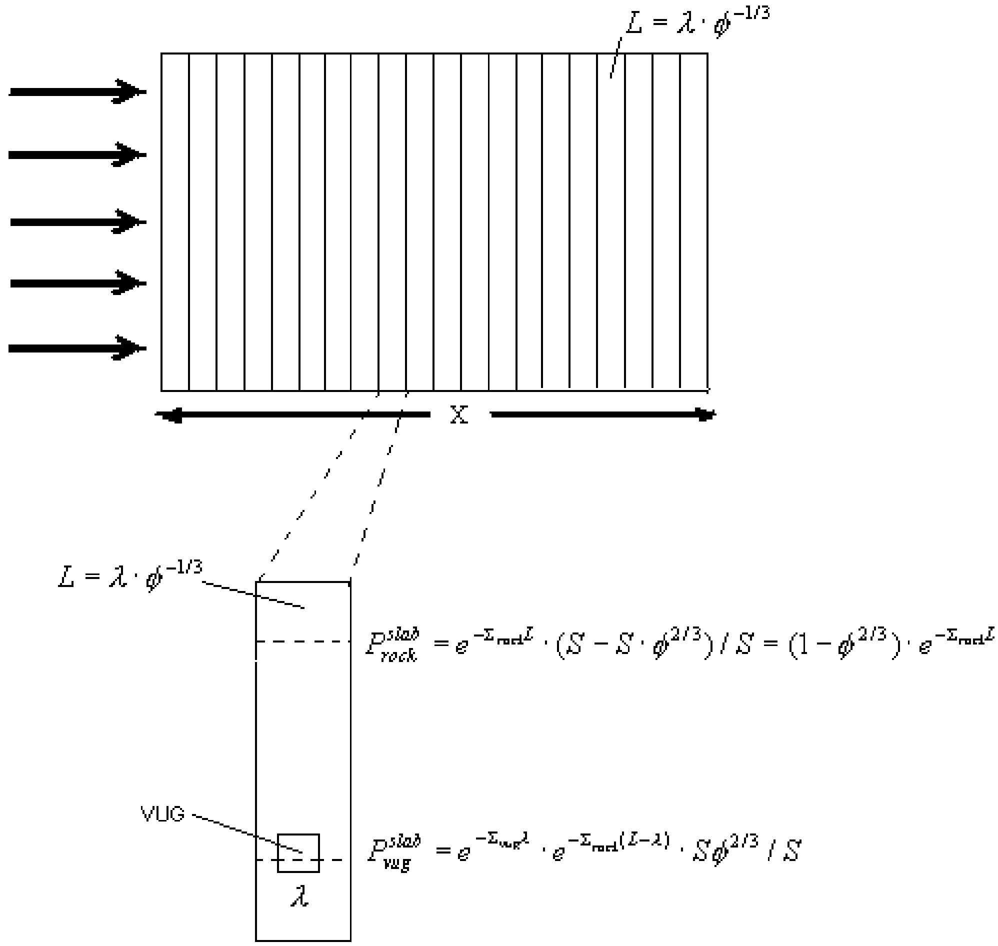
Transmission Probability Method for a Pair of Laminated Vuggy Porous Media

The algebraic properties of (14) have been explored in some detail – it remains borderline tractable. However, when transmission probabilities for laminae with two pore systems are developed analogous to equations (10-13), it has not been possible to obtain an expression for Σ_{HET} analogous to equation (14) in closed form. Instead, computer software has been utilized to continue development of the Transmission Probability Method to laminae and other more complex pore geometries. Figures 2 and 3 show the main geometrical features for both the parallel and perpendicular laminar cases.

Note that for the case of parallel laminae, TPM assumes that the associated linear dimension of Material2 is less that that for Material1. With reference to Figure 2 for the parallel laminae, TPM computes the transmission probability to a macroscopic distance **X** by summing the probabilities through the individual L_2 type slabs, i.e. the neutrons/gamma rays must pass through slab1 **and** slab2 … **and**…**and** slab n. However, at each step in each of these L_2 type slabs, the neutrons/gamma rays can penetrate the matrix **or** pore of Material1 **or** the matrix **or** pore of Material2, etc. It is the unique way in which TPM performs this transmission probability calculation that distinguishes it from the standard approach detailed above. The exact algebraic/software details for both the parallel and perpendicular laminated cases are not provided at this time. Instead, examples are provided comparing the results of the standard and newer LVPM computations.

Figure 1

Neutron rock and vug transmission probabilities in a vuggy porous medium through (x/L) micro slabs each of thickness L to a macro distance x



$$P_{HET}^{TRANS} = [P_{HET}^{slab}]^{x/L} = e^{-\Sigma_{rock} x}[1 - \phi^{2/3}(1 - e^{-(\Sigma_{vug} - \Sigma_{rock})\lambda})]^{x/L}$$

$$\Sigma_{HET} = \Sigma_{rock} - (\phi^{1/3} / \lambda)\ln[1 - \phi^{2/3}(1 - e^{-(\Sigma_{vug} - \Sigma_{rock})\lambda})]$$

Figure 2

Neutron transmission in 2 parallel laminated vuggy porous media 1 & 2 through (x / L_2) micro slabs each of thickness L_2 to a macro distance x

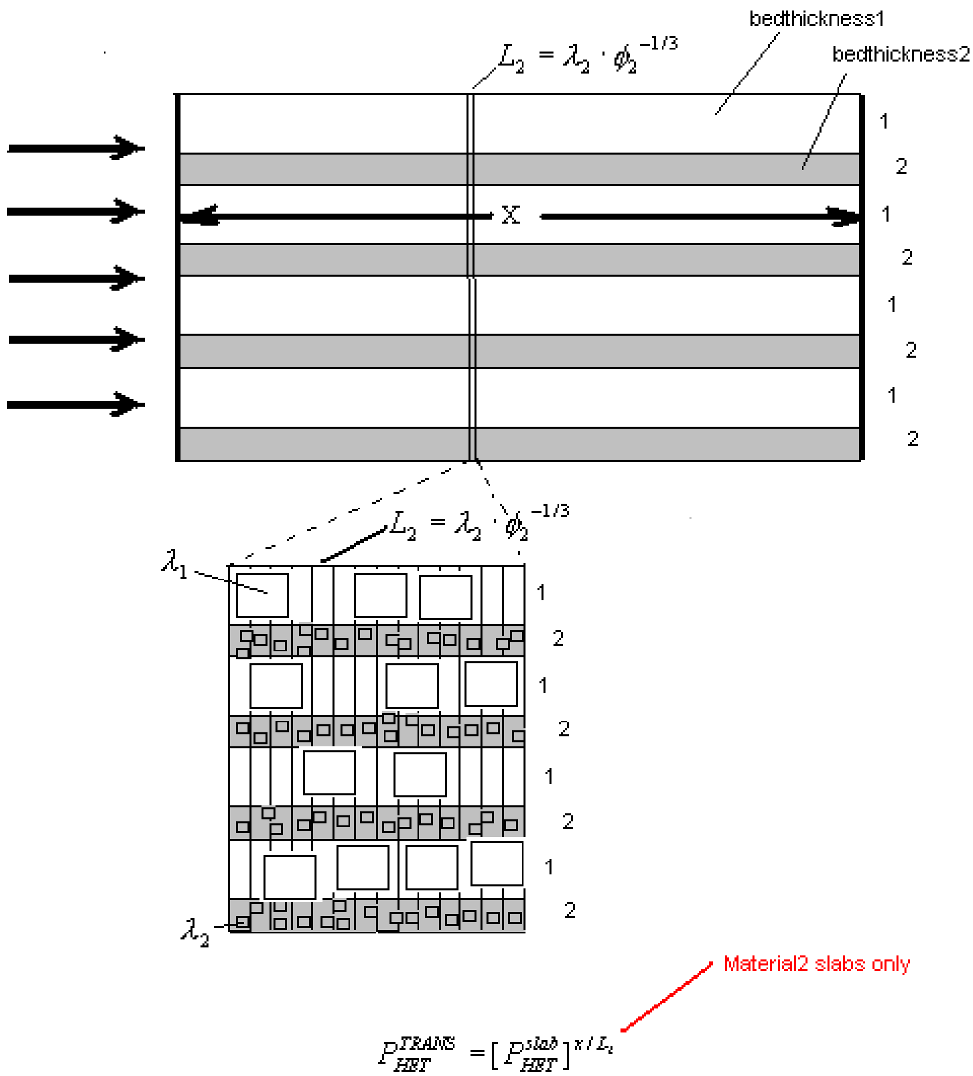


Figure 3

Neutron transmission in 2 perpendicular laminated vuggy porous media 1 & 2 through (x_1 / L_1) micro slabs each of thickness L_1 and (x_2 / L_2) micro slabs each of thickness L_2 to a total macro distance $x = n \cdot (x_1 + x_2)$